Re-explain location choices: income, transportation, and lifestyle change

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Abstract

Classic studies on urban economics, specifically the mono-centric model, have relied heavily on the least-cost method and the utility maximization approach. However, self-restricted by the unknown nature of the household utility function, it seems that without altering the existing framework, it is hard to make further theoretical implications. To make improvements on the classic mono-centric model so it can be more practical, a new analytical approach may be needed. Growing from the roots of traditional methods, this paper presents alternatively a general theoretical framework, which can be used to extend the theoretical implications of the mono-centric model more precisely in terms of income, transportation, and lifestyle change. By using a return rate approach, the paper is trying to simplify the complex mathematic equations used by previous theories on the role of income and transportation. It is also demonstrated that this framework has capability to take lifestyle change as an extra factor for location choices.
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Introduction

This paper presents a general theoretical framework, which can be used to explain urban location choices in terms of income, transportation, and lifestyle change. By using a return rate based method, the paper is trying to simplify the complex mathematic equations used by previous theories on the role of income and transportation. Later, it is also demonstrated that this framework has capability to take lifestyle change as an extra factor for location choices.

Location choices have been characterized as a decision affected by two major economic factors: income and transportation. With increased transportation cost being a major obstacle, it has been said that if the greater demand for housing $q$ induced by income increase can overcome the transportation obstruction $t$, the rich may then move to suburban locations (Wheaton, 1977, Leroy and Sonstelie, 1983, Brueckner, Thisse, and Zenou, 1999). This statement, derived based on the standard mono-centric model developed by Alonso, Muth, and Mills and later summarized by Fujita and Krugman (Alonso, 1964; Muth, 1969; Mills, 1967 and 1972; Fujita, 1989; Fujita and Krugman, 1995) is empirically very true, but it has been derived through complex mathematical equations. Previous theories focus on the condition of $t/q$ ($t$: commuting cost per mile, $q$: housing consumption). As both $t$ and $q$ can increase as income increases, the value $t/q$ may either increase or decrease when income increases. If the relationship is positive, the bid rent curve for the rich is steeper and therefore the rich would prefer to live in central locations. Otherwise, the rich would move to suburban locations. It seems that although the relationship between transportation cost and income increase is predictable using a time-wage approach, it is difficult to quantify the $q$ increase in terms of income increase. The problem is caused by a possible reason. In the maximization model, other goods $e$ was substituted by $q$ into the utility function based on the budget equation. Since the exact utility substitution function is unknown, it is difficult to predict the exact relationship between income increase and $q$ increase. The equation for the first order condition defines only a positive relationship between income increase and $q$ increase. The equation contains unknown functions with $q$ as a variable, so it is hard to quantify the relationship between income increase and $q$ increase without knowing the exact utility function $u(e, q)$. As a result, previous studies have been focusing more on empirical test of the $t/q$ value. Even though many empirical tests show positive relationships between income increase and suburbanization, for example, Bradford and Kelejian (1973), Steinnes and Fisher (1974), Grubb (1982), and Margo (1992), it seems that these empirical results do not serve as evidence supporting any theory since the theory itself lacks the capability in predicting these empirical results meaningfully. The question now
is, is it possible to predict location choices theoretically and precisely in terms of income, transportation, and other factors?

Nevertheless, buried deep in mathematical equations, there may still be fundamental meanings that can be retrieved to remind us the true reasoning force behind, without knowing which it seems that the traditional analysis has reached a sophisticated dead end. This study, using a different framework, is trying to provide general predictions to answer the questions unanswered by traditional methods theoretically. In fact, previous studies share exactly the same concept used here: as the least-cost method is equivalent to the concept of return rate used here.

The least-cost method is defined as,

“…graphically as the tangency point between the relevant isoquant and an isocost line (Lipsey, 1975, pp230)”

**Proposition:** If we define return rate as the value of utility at the above tangency point divided by the value of the cost line, the return rate would be maximized for any given cost line exactly at the tangency point, in which sense the return rate approach is equivalent to the least-cost method.

The proposition can be easily proved logically: the tangency point gives the maximum utility, so with the numerator (utility) being maximized and the denominator (cost) remaining the same, the return rate is maximized at the same point as well. With the return rate approach, the traditional framework can be reformatted and become more flexible in explaining urban dynamics. The method starts with a consumption ratio \( x \), which is the total amount of housing consumption \( h \) divided by the total amount of consumption of other goods \( e \). If the price for one unit of \( e \) is \( p \) and the rent level is \( r \), then the cost for the combination of each unit of \( e \) plus \( x \) units of housing is then \( p + rx \). The money value of utility for this small combination package is then \( u(x) \). With these two functions, we can then reformat the problem in the following section.

**The reformatted framework**

Suppose \( B \) is the total budget and \( T \) is the transportation cost. The units of combination package consumed is then,

\[
y = \frac{B - T}{p + rx}
\]

To maximize the utility given a variable budget \( B \) then equals to maximize the total return rate in equation 2 with variable \( x \) and \( y \) and parameters \( B, T, p, \) and \( r \),

\[
\text{Max } \psi(x) = \frac{yu(x) - B}{B}
\]
Substitute 1) into 2), equation 2 can be further simplified as below with \( x \) being the only variable,

\[
\psi(x) = \frac{B - T}{B} \frac{u(x)}{p + r x} - 1
\]

Max 3)

To solve the problem we have \( \varphi'(x) = 0 \), so we have,

\[
\frac{u'(x)}{u(x)} = \frac{r}{p + r x}
\]

4)

So \( x^* = f(p, r) \), showing that to maximize the total return rate the optimal size of housing consumption combined with each unit of other goods is a function of the two prices for housing and other goods. In other words, maximization of the total return rate equals maximization of each package’s return rate.

The optimal return rate \( \varphi^*(x) \) is then,

\[
\psi^*(x) = \frac{u'(x^*)}{r} \frac{B - T}{B} - 1
\]

5)

Now if the household is to choose a location to live, the rent he is willing to pay should make the return rate indifferent when \( T \) varies over space. Thus, we have,

\[
\frac{u'(x^*)}{r} \frac{B - T}{B} = C
\]

6)

In equation 6, \( C \) is a positive constant (\( C > 1 \)) and \([u'(x^*)/r]\) is the optimal return rate for each package. \([(B-T)/B]\) is a discount function showing that the return rate is discounted by distance. Since \( B - T \) decreases as distance increases, the household may select an appropriate rent value to increase \( u'(x^*)/r \) so that the constant total return rate can be maintained.

The relationship between \( r \) and \( u'(x^*)/r \) can be predicted using Graph 1 below,
In Graph 1, the optimal x value is the point at which the two curves \( \frac{u'(x)}{u(x)} \) and \( \frac{r}{p+rx} \) cross with each other. If r increases from \( r_0 \) to \( r_1 \), the curve \( \frac{r}{p+rx} \) shifts up and the resulting optimal x value decreases from \( x_0^* \) to \( x_1^* \). The optimal return rate for one package \( \frac{u'(x^*)}{r} \) decreases with r since the cost line becomes steeper. Thus when distance increases, the bid rent value has to decrease in order to maintain the constant total return rate over space.

\[
\frac{u'(x^*)}{r} = \frac{C'}{1 - \frac{T}{B}}.
\]

Equation 6 can be rewritten as the condition \( \frac{u'(x^*)}{r} = \frac{C'}{1 - \frac{T}{B}} \). The bid rent curve can then be derived using Graph 2 below.
In Graph 2, the first quadrant shows the bid rent curve (r is rent and d is distance). In the second quadrant, there is a 45-degree mirror to project the curve in the third quadrant into the first quadrant. The third quadrant shows the curve $u'(x^*)/r$ (v is simply numeric value or optimal package return rate). The fourth quadrant shows the curve $C/(1-T/B)$. When distance is zero, $C/(1-T/B)$ cuts the v axis at C. The equation $u'(x^*)/r = C/(1-T/B)$ stands, so $u'(x^*)/r$ also equals C (no discount), which gives the bid rent $r_0$. When distance increases to $d_1$ (a suburban location), the bid rent becomes $r_1$, which is lower than $r_0$ and gives a downward bid rent curve as distance increases. If income increases (B increases), the curve $C/(1-T/B)$ turns up anti-clockwise to $C/(1-T/B_1)$ using the point C on the v axis as a pivot. As a result, for the same suburban location $d_1$, the bid rent $r_1$ increases to $r_2$, which gives a flatter bid rent curve. On the contrary, if $T$ increases for $d_1$, the curve $C/(1-T/B)$ turns down clockwise, indicating a steeper bid rent curve. Thereby, for normal cases, the steepness of bid rent curves is merely determined by the steepness of the curve $C/(1-T/B)$.

To analyze further the substitution effect between income and transportation, the above analysis shows that the term $T/B$ may be the key. For each distance $d$, if $T/B$ decreases, the bid rent curve becomes flatter. If $T/B$ increases, the bid rent curve becomes steeper. $T/B$ can be decomposed further into several versions. If the household has wage income only, $T/B$ can be written as below,

$$ \frac{(t + \frac{w}{s})d}{whr} $$

In the formula above, $w$ is the hourly wage, $t$ is the fixed travel cost per mile, $s$ is the speed of travel, $d$ is the distance, and $hr$ is the working hours. It can be further written as,

$$ \frac{(\frac{t}{w} + \frac{1}{s})d}{hr} $$

It shows that if the travel speed is not changed, then for each distance $d$, $T/B$ decreases as $w$ increases. The bid rent curve will become flatter. If the rich use automobile to travel, $t$ increases and $s$ also increases, whether the $T/B$ value increases with $w$ or not then depends on the actual value of $t$, $w$, and $s$. If automobile is so efficient that the $s$ increase is much larger than the $t$ increase, then $T/B$ may be significantly reduced, implying a much flatter bid rent curve.

If the household has both wage income and asset income, $T/B$ can be written as the second version,

$$ \frac{(t + \frac{w}{s})d}{whr + A} $$

In the formula, $A$ is asset income. It can be further written as,
In this version, an increase in asset income can make T/B smaller, implying a flatter bid rent curve. If w increases, differentiate f(w), we have,

\[
f'(w) = \frac{(\frac{1}{w} + \frac{1}{s})d}{hR + \frac{A}{w}}.
\]

If A/s - t\times hr > 0, the bid rent curve becomes steeper. If A/s - t\times hr < 0, the bid rent curve becomes flatter. If both A and w increase, then we will have to conduct detailed analysis case by case.

The third version deals with the amenity level over space. If the household visits amenity sites in downtown area frequently, then comparing to other households who do not, moving to a remote location makes the composite travel cost T larger, in which case an income increase may not be able to flatten the bid rent curve, as what happens in Paris and many European cities.

In addition to the analysis of T/B, it is also possible to alter the curve \(u'(x^*)/r\). If the household tends to consume more goods outside (shopping, watching movie, going to a park, etc), the \(u(x)\) curve may become more concave, generating a flatter \(u'(x^*)/r\) curve (for the same \(r\) value, the value \(u'(x^*)/r\) increases). The bid rent curve then becomes steeper, indicating the household is more likely to live in central locations. On the contrary, if the household tends to consume more at home (large appliance, home pool, yard, etc), then the \(u(x)\) curve may become less concave, generating a steeper \(u'(x^*)/r\) curve and hence a flatter bid rent curve.

**Conclusion**

In summary, this paper analyzed the mechanism behind location choices using a return rate approach. The paper confirmed the empirical results from previous studies on the role of income increase on location choices. By simplifying previous methods, it summarizes three types of causes that may alter the bid rent curve and cause suburbanization. These include income change, transportation change, and lifestyle change. Each type has also been decomposed into several scenarios and detailed analyses on each one were conducted.

The result has both theoretical and empirical meanings. On the one hand, it provides a simple way to predict location choices theoretically. On the other, the parameters used in this study define certain conditions for further empirical tests. While the actual situation may violate the theoretical premise due to exceptions, the analysis may still be capable in shedding lights on the urban economics knowledge base through a very simple way to integrate both theoretical and empirical analysis.
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